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WAVE FLOWS IN A THIN LAYER OF A VISCOUS LIQUID. INFLUENCE OF A CONSTANT ELECTRIC FIELD

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The influence of a constant transverse electric field on the dynamics of longwave, weakly nonlinear flow of a viscous dielectric liquid film down a vertical wall is studied. An amplitude integrodifferential equation in partial derivatives of the Kuramoto–Sivashinskii equation type, which describes the behavior of the free surface of the layer, is derived using the method of multiscale stretching. In the case considered, the potential energy of the electric field is a source of longwave perturbations, but, on the whole, secondary regimes are apparently nonlinearly steady. Probably, the electric polarization effects studied can be used as a factor that governs the dynamics of film flow.

Introduction. Investigation of various flows of thin layers of a viscous liquid is of interest both in theory (because the dynamics in these rather simple and accessible physical systems is extremely varied and involves many important nonlinear phenomena [1–3]) and in practice (for example, for activation of transfer processes in heat- and mass-exchange devices [4, 5]).

The behavior of a thin liquid layer (a film) is frequently studied by deriving amplitude equations in a longwave approximation. A typical example is the Kuramoto–Sivashinskii equation (KS), used to describe weakly nonlinear processes in film layers within the limit of asymptotically large effective surface tension [6–10]. The main physical factors in this case are the gravitational force, viscous friction, and surface tension; predominance of effects due to the gravitational force over viscosity effects facilitates destabilization of longwave perturbations, and surface tension results in attenuation of small shortwave fluctuations in the linear stage. The mutual effect of these two opposite tendencies leads to the complex and varied dynamics expressed by the KS equation [6–15].

In the present paper, we examine how a normal constant electric field can affect the behavior of a film.

1. Formulation of the Problem. Let x^* and z^* be Cartesian coordinates (we restrict ourselves to the two-dimensional case) and the z^* axis be directed opposite to the direction of the gravitational force. We study the following physical system (Fig. 1). The surfaces $x^* = 0$ and $x^* = a^* = \text{const} > 0$ define the planes of electrodes between which in the region $\Omega = \{0 < x^* < h^*(z^*, t^*),\ -\infty < z^* < \infty\}$ flow of a thin film layer of a viscous incompressible dielectric liquid occurs. In the region $\Omega^g = \{h^*(z^*, t^*) < x^* < b^*,\ -\infty < z^* < \infty\}$ ($b^* = \text{const}$ and $b^* < a^*$), there is a dielectric gas present, and in the region $\Omega^s = \{b^* < x^* < a^*,\ -\infty < z^* < \infty\}$, there is a solid dielectric present. We assume that the primary flow is laminar, the unperturbed free boundary of the film layer is plane and is defined by the equation $x^* = h_0^*$, the electric field between the electrodes is constant and homogeneous, and the electric intensity of the thin liquid layer Ω is equal to e_0^*. It is assumed that $h_0^*/a^* \ll 1$, $(b^* - h_0^*)/a^* \ll 1$, and $(a^* - b^*)/a^* \sim O(1)$.

The following (constant) parameters enter the problem: ρ, μ, and γ are the density, dynamic viscosity, and dielectric constant of the liquid film layer Ω; γ^g and γ^s are the dielectric permeability of the phases in the regions Ω^g and Ω^s; σ is the surface-tension coefficient at the interface; and f^* is the acceleration of gravity.

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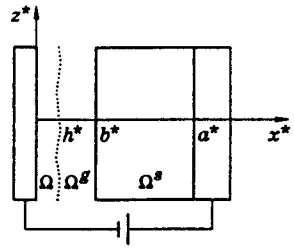

Fig. 1

As scale factors for spatial variables, velocity, time, pressure, and electric intensity, we use, respectively, h_0^*, $w^* = \rho f^* h_0^{*2}/2\mu$, which is the velocity on the free boundary of the film, h_0^*/w^*, ρw^{*2}, and e_0^*. Next we set $(x^*, y^*, a^*, b^*, h^*)/h_0^* = (x, y, a, b, h)$, $t^* w^*/h_0^* = t$ and examine the dimensionless problem.

Let p be the pressure, u and w be the x and z components of the film velocity vector, and e and g, e^g and g^g, and e^s and g^s be the x and z components of the electric-intensity vector in the regions Ω, Ω^g, and Ω^s, respectively.

The initial mathematical model that describes the processes in the physical system studied has the following form [16, 17]. In the region Ω, the Navier–Stokes, continuity, and Maxwell equations are satisfied (a magnetic field is absent):

$$u_t + uu_x + wu_z = -p_x + (1/\mathrm{Re})(u_{xx} + u_{zz}); \tag{1.1}$$

$$w_t + uw_x + ww_z = -p_z + (1/\mathrm{Re})(w_{xx} + w_{zz}) - 2/\mathrm{Re}; \tag{1.2}$$

$$u_x + w_z = 0; \tag{1.3}$$

$$e_z - g_x = 0, \quad e_x + g_z = 0. \tag{1.4}$$

In the region Ω^g, we have

$$e_z^g - g_x^g = 0, \quad e_x^g + g_z^g = 0 \tag{1.5}$$

and in the region Ω^s, we have

$$e_z^s - g_x^s = 0, \quad e_x^s + g_z^s = 0. \tag{1.6}$$

The conditions on the electrodes are of the form

$$u = 0,\ w = 0,\ g = 0 \quad \text{for } x = 0 \quad \text{and} \quad g^s = 0 \quad \text{for } x = a. \tag{1.7}$$

The conditions on the interface Γ [at $x = h(z,t)$] express (the notation $[(.)] \equiv (.) - (.)^g$ is used):

— continuity of the electric-induction vector component normal to Γ

$$[\chi(e - h_z g)] = 0; \tag{1.8}$$

— continuity of the electric-intensity vector component tangent to Γ

$$[eh_z + g] = 0; \tag{1.9}$$

— balance of normal stresses

$$-\mathrm{Re}(p - p^g)(1 + h_z^2) + [\chi e^2] + h_z^2[\chi g^2] - 2h_z[\chi eg] - (1/2)(1 + h_z^2)[\chi(e^2 + g^2)] + 2(u_x - h_z(w_x + u_z) + h_z^2 w_z) = \mathrm{We} h_{zz}/(1 + h_z^2)^{1/2}; \tag{1.10}$$

— balance of tangential stresses

$$h_z[\chi e^2] + (1 - h_z^2)[\chi eg] - h_z[\chi g^2] + 2h_z u_x + (1 - h_z^2)(w_x + u_z) - 2h_z w_z = 0; \tag{1.11}$$

— the kinematic relation

$$h_t + wh_z = u. \tag{1.12}$$

For $x = b$, the following conditions are satisfied:

$$\chi^g e^g = \chi^s e^s; \tag{1.13}$$

$$g^g = g^s. \tag{1.14}$$

Here $\mathrm{Re} = \rho h_0^* w^*/\mu$, $\mathrm{We} = \sigma/(\mu w^*)$, $\chi = \gamma h_0^* e_0^{*2}/(4\pi\mu w^*)$, $\chi^g = \gamma^g h_0^* e_0^{*2}/(4\pi\mu w^*)$, $\chi^s = \gamma^s h_0^* e_0^{*2}/(4\pi\mu w^*)$, and p^g is the pressure of the gas phase in contact with the film layer (a given quantity).

The principal state is defined by the following solution of system (1.1)–(1.14):

$$\begin{gathered} u_0 = 0, \quad w_0 = x^2 - 2x, \quad p_0 = \text{const}, \quad h = 1, \quad e_0 = \delta, \quad g_0 = 0, \\ e_0^g = (\chi/\chi^g)\delta, \quad g_0^g = 0, \quad e_0^s = (\chi/\chi^s)\delta, \quad g_0^s = 0, \quad \text{where } \delta \pm 1. \end{gathered} \tag{1.15}$$

2. Derivation of the Amplitude Equation. In deriving the amplitude equation we follow the approaches of [7, 8], which implement the method of multiscale decomposition. If the influence of the electric field is ignored, the examined film flow will be unsteady within the framework of linear analysis. In this case, according to the dispersion relation for perturbations of the form $\exp(\lambda t + ikz)$, for $\mathrm{We} \gg 1$ and $\mathrm{Re} \sim O(1)$ in the instability region, we have [8]

$$k \sim \mathrm{We}^{-1/2}, \quad \mathrm{Real}\,(\lambda) \sim \mathrm{We}^{-1}, \quad \mathrm{Im}\,(\lambda) \sim \mathrm{We}^{-1/2}. \tag{2.1}$$

Thus, one might expect that, inside the film under rather strong surface tension, the characteristic longitudinal spatial scale of weakly nonlinear secondary wave regimes will be much larger than the scale in the transverse direction when electric effects do not significantly affect the situation described above.

Linearizing system (1.1)–(1.14) on the solution (1.15) and expanding the quantity λ in a series in the asymptotically small wave number k, we find that estimates (2.1) remain valid if the conditions $\chi = O(1)$, $\chi^g = O(1)$, and $\chi^s = O(\mathrm{We}^{1/2})$ are satisfied.

Next, we consider the behavior of secondary processes with a characteristic spatial scale of order $O(\varepsilon^{-1})$ ($\varepsilon \ll 1$ is a small parameter). We assume in this case that $\mathrm{We} = O(\varepsilon^{-2})$.

Taking into account (2.1), we set

$$\mathrm{We} = O(\varepsilon^{-2}), \quad \mathrm{Re} = O(1), \quad \chi = O(1), \quad \chi^g = O(1), \quad \chi^s = O(\varepsilon^{-1}) \quad (\chi^g \neq \chi^s); \tag{2.2}$$

$$\begin{gathered} X = x \quad (0 \leqslant x \leqslant 1), \qquad R = (b - x)/(b - 1) \quad (1 \leqslant x \leqslant b), \\ Y = (a - x)/(a - b) \quad (b \leqslant x \leqslant a), \qquad Z = \varepsilon z, \qquad \frac{\partial}{\partial t} \to \varepsilon \frac{\partial}{\partial \tau} + \varepsilon^2 \frac{\partial}{\partial T}; \end{gathered} \tag{2.3}$$

$$\begin{gathered} u = \sum_{n=1} \varepsilon^n U_n(X, Z, \tau, T), \qquad w = w_0(X, \varepsilon) + \sum_{n=1} \varepsilon^n W_n(X, Z, \tau, T), \\ p = p_0(\varepsilon) + \sum_{n=1} \varepsilon^n P_n(X, Z, \tau, T), \qquad h = 1 + \sum_{n=1} \varepsilon^n H_n(Z, \tau, T), \\ e = \delta + \sum_{n=1} \varepsilon^n E_n(X, Z, \tau, T), \qquad g = \sum_{n=1} \varepsilon^n G_n(X, Z, \tau, T), \\ e^g = (\chi/\chi^g)\delta + \sum_{n=1} \varepsilon^n E_n^g(R, Z, \tau, T), \qquad g^g = \sum_{n=1} \varepsilon^n G_n^g(R, Z, \tau, T), \\ e^s = (\chi/\chi^s)\delta + \sum_{n=1} \varepsilon^n E_n^s(Y, Z, \tau, T), \quad g^s = \sum_{n=1} \varepsilon^n G_n^s(Y, Z, \tau, T). \end{gathered} \tag{2.4}$$

In addition, we denote $\overline{\mathrm{We}} = \mathrm{We}\,\varepsilon^2$, $\overline{\chi}^s = \chi^s \varepsilon$, and $\alpha = \varepsilon a$.

Substituting (2.2)–(2.4) into (1.1)–(1.14), in the zero and first orders we obtain

$$-P_{1X} + (1/\mathrm{Re})U_{1XX} = 0, \quad (1/\mathrm{Re})W_{1XX} = 2(X - 1)U_1, \quad U_{1X} = 0,$$

$$G_{1X} = 0, \quad E_{1X} = 0 \quad \text{for } 0 < X < 1; \tag{2.5}$$

$$G^g_{1R} = 0, \quad E^g_{1R} = 0 \quad \text{for } 0 < R < 1; \tag{2.6}$$

$$E^s_{1Z} + \frac{1}{\alpha} G^s_{1Y} = 0, \quad -\frac{1}{\alpha} E^s_{1Y} + G^s_{1Z} = 0 \quad \text{for } 0 < Y < 1; \tag{2.7}$$

$$U_1 = W_1 = G_1 = 0 \quad \text{for } X = 0, \quad G^s_1 = 0 \quad \text{for } Y = 0; \tag{2.8}$$

$$\begin{gathered} E_1 = (\chi^g/\chi)E^g_1, \quad G_1 - G^g_1 = 0, \quad -\mathrm{Re}\, P_1 + \chi\delta(E_1 - E^g_1) + 2U_{1X} = \mathrm{We}\, H_{1ZZ}, \\ \chi\delta(G_1 - G^g_1) + W_{1X} + 2H_1 = 0, \quad U_1 = 0 \quad \text{for } X = 1, \ R = 1; \end{gathered} \tag{2.9}$$

$$E^s_1 = 0, \quad G^g_1 - G^s_1 = 0 \quad \text{for } R = 0, \ Y = 1. \tag{2.10}$$

Furthermore, we write the condition necessary for solution of the problem in the second order:

$$E^g_1 = (\overline{\chi}^s/\chi^g)E^s_2 \quad \text{for } R = 0, \ Y = 1. \tag{2.11}$$

Problem (2.5)–(2.10) has the solution

$$\begin{gathered} U_1 = 0, \quad P_1 = -(\overline{\mathrm{We}}/\mathrm{Re})H_{1ZZ} + (\chi\delta/\mathrm{Re})(E_1 - E^g_1), \quad W_1 = -2H_1X, \\ E_1 = E_1(Z,\tau,T), \quad G_1 = 0, \quad E^g_1 = (\chi/\chi^g)E_1(Z,\tau,T), \quad G^g_1 = 0, \quad E^s_1 = 0, \quad G^s_1 = 0. \end{gathered} \tag{2.12}$$

The function $E_1(Z,\tau,T)$ is found as follows.

To determine the amplitude equation, it is necessary to examine Eqs. (1.2)–(1.7), (1.9), (1.11), (1.12), and (1.14) in the second order:

$$W_{1\tau} + 2(X-1)U_2 + X(X-2)W_{1Z} = -P_{1Z} + (1/\mathrm{Re})W_{2XX} \quad \text{for } 0 < X < 1; \tag{2.13}$$

$$U_{2X} + W_{1Z} = 0 \quad \text{for } 0 < X < 1; \tag{2.14}$$

$$E_{1Z} - G_{2X} = 0, \quad E_{2X} = 0 \quad \text{for } 0 < X < 1; \tag{2.15}$$

$$(1-b)E^g_{1Z} - G^g_{2R} = 0, \quad E^g_{2R} = 0 \quad \text{for } 0 < R < 1; \tag{2.16}$$

$$E^s_{2Z} + \frac{1}{\alpha} G^s_{2Y} = 0, \quad -\frac{1}{\alpha} E^s_{2Y} + G^s_{2Z} = 0 \ \text{ for } 0 < Y < 1, \ \ G^s_2 = 0 \ \text{ for } Y = 0; \tag{2.17}$$

$$U_2 = W_2 = G_2 = 0 \quad \text{for } X = 0; \tag{2.18}$$

$$\delta(1 - \chi/\chi^g)H_{1Z} + G_2 - G^g_2 = 0 \quad \text{for } X = 1, \ R = 1; \tag{2.19}$$

$$\chi(1 - \chi/\chi^g)H_{1Z} + \chi\delta(G_2 - G^g_2) + W_{2X} + 2H_2 = 0 \quad \text{for } X = 1, \ R = 1; \tag{2.20}$$

$$H_{1\tau} - H_{1Z} = U_2 \quad \text{for } X = 1; \tag{2.21}$$

$$G^g_2 = G^s_2 \quad \text{for } R = 0, \ Y = 1. \tag{2.22}$$

From (2.12), (2.14), and (2.18) we obtain

$$U_2 = X^2 H_{1Z}. \tag{2.23}$$

Then, from (2.21) it follows that $H_1 = H_1(\zeta, T)$ and $\zeta = Z + 2\tau$. From (2.12), (2.15), (2.16), (2.18) and (2.22) it follows that

$$G_2 = E_{1\zeta}X, \quad G^g_2 = (\chi/\chi^g)(1-b)E_{1\zeta}R + G^s_2(Y=1). \tag{2.24}$$

We define the function Φ by the relations $E^s_{2\zeta} = \Phi_Y$ and $G^s_2 = -\alpha\Phi$. Then using the Fourier transform $[F(\Phi) = \int\limits_{-\infty}^{\infty} \Phi \exp(-ik\zeta)\, d\zeta]$, from (2.17) we find

$$F(\Phi) = C(k)\{\exp(\alpha k Y) - \exp(-\alpha k Y)\},$$

and from (2.11), (2.12), (2.19), and (2.24) we have

$$E_{1\zeta} = (\overline{\chi}^s/\chi)\Phi_Y, \quad \delta(1 - \chi/\chi^g)H_{1\zeta} + (1 + (\chi/\chi^g)(b-1))E_{1\zeta} + \alpha\Phi = 0 \quad \text{for } Y = 1.$$

Hence,

$$E_{1\zeta} = F^{-1}\Big(\frac{\delta\overline{\chi}^s k(\chi - \chi^g)F(H_{1\zeta})}{\chi\chi^g \tanh(\alpha k) + \overline{\chi}^s k(\chi^g + \chi(b-1))}\Big). \tag{2.25}$$

The quantity W_2 is determined from (2.13), (2.18), and (2.20) using (2.12), (2.19), and (2.23):

$$W_2 = \operatorname{Re}\Big(\frac{1}{6}X^4 - \frac{2}{3}X^3 + \frac{4}{3}X\Big)H_{1\zeta} - \frac{1}{2}\overline{\mathrm{We}}X(X-2)H_{1\zeta\zeta\zeta} - 2XH_2 + \frac{1}{2}\chi\delta X(X-2)\Big(1 - \frac{\chi}{\chi^g}\Big)E_{1\zeta}. \tag{2.26}$$

Examining in the third order the kinematic condition

$$H_{1T} + H_{2\zeta} + W_1 H_{1\zeta} = U_3 + U_{2X}H_1 \quad \text{for } X = 1$$

and expressing the function U_3 from the continuity equation $U_{3X} + W_{2\zeta} = 0$ with allowance for $U_3(0) = 0$ and (2.25) and (2.26), we find the desired amplitude equation in the form

$$H_{1T} - 4H_1H_{1\zeta} + \frac{8}{15}\operatorname{Re} H_{1\zeta\zeta} + \frac{1}{3}\overline{\mathrm{We}}H_{1\zeta\zeta\zeta\zeta} + \frac{i}{2\pi}\int_{-\infty}^{\infty} J(k)\int_{-\infty}^{\infty} H_{1\xi}(\xi,T)\exp(ik(\zeta-\xi))\,d\xi\,dk = 0,$$

$$J(k) = \frac{Dk^2}{\tanh(\alpha k) + Ek}, \quad D = \frac{\overline{\chi}^s}{3}\Big(\frac{\chi}{\chi^g} - 1\Big)^2, \quad E = \frac{\overline{\chi}^s}{\chi}\Big(1 + (b-1)\frac{\chi}{\chi^g}\Big). \tag{2.27}$$

Remark. Formally, it is possible to examine the case where the region Ω^s, like the region Ω^g, is filled with a gas phase (the results will be similar) but, in this situation, the question arises of how to realize the condition $\gamma \ll \gamma^s$, which is key to our analysis, in practice.

3. Influence of an Electric Field on the Properties of Periodic Regimes. Weakly nonlinear processes in the studied system in the absence of a potential difference between the electrodes are described by a KS equation that is similar in form to (2.27), but one should set $J \equiv 0$. We examine how the electric effects studied can change the dynamics of the film layer.

With substitution of variables, Eq. (2.27) with periodic boundary conditions $[H_1(0,t) = H_1(L,t)]$ can be written as

$$\frac{\partial H}{\partial t} + \mu^2\frac{\partial^2 H}{\partial x^2} + \frac{\partial^4 H}{\partial x^4} + H\frac{\partial H}{\partial x} + \frac{i}{2\pi}\int_{-\infty}^{\infty} J(k)\int_{-\infty}^{\infty}\frac{\partial H}{\partial \xi}\exp(ik(x-\xi))\,d\xi\,dk = 0,$$

$$H(0,t) = H(2\pi,t),\ \mu^2 = \frac{8}{5}\frac{\operatorname{Re}}{\overline{\mathrm{We}}}\Big(\frac{L}{2\pi}\Big)^2,\ J(k) = \frac{Ak^2}{\tanh(\sigma k) + Bk},\ A > 0,\ B > 0,\ \sigma > 0. \tag{3.1}$$

We compare the periodic solutions described by Eq. (3.1) and the KS equation

$$\frac{\partial H}{\partial t} + \nu^2\frac{\partial^2 H}{\partial x^2} + \frac{\partial^4 H}{\partial x^4} + H\frac{\partial H}{\partial x} = 0. \tag{3.2}$$

Expanding the solution in the Fourier series $H = \sum\limits_{-\infty}^{\infty} A_n(t)\exp(inx)$, $A_{-n} = A_n^{\text{c.c.}}$ (the superscript c.c. means complex conjugation), we find that the evolution of the amplitudes of the Fourier components is described by the following system of ordinary differential equations:

$$\frac{dA_n}{dt} = \gamma(n)A_n - in\Big(\sum_{r=1}^{\infty} A_r^{\text{c.c.}}A_{n+r} + \frac{1}{2}\sum_{r=1}^{n-1} A_r A_{n-r}\Big), \quad n = 1, 2, \ \ldots. \tag{3.3}$$

Here $\gamma(k) = \mu^2k^2 - k^4 + kJ(k)$ and $k \in R$. (By virtue of the fact that $\gamma(0) = 0$ the coefficient A_0 is constant and is equated to zero.)

The structure of the linear dispersion relation is of fundamental importance for the KS equation, as well as for other weakly nonlinear systems. As can be seen from (3.3), for the KS equations, all Fourier modes

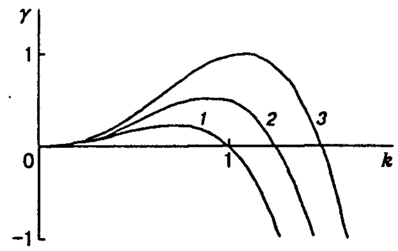

Fig. 2

damp in the linear stage if $n > \nu$. Thus, the number of linearly unstable modes is finite, and the KS equation itself is shown [12, 13] to be equivalent to a finite dynamic system of ordinary differential equations.

For relatively small values of the parameter ν, the number of linearly unstable Fourier harmonics is small, and the limiting regimes of the KS equation are ordered oscillatory or steady states. The latter are called j-modal steady states and can be written (with accuracy to a phase shift) as [12]

$$H_j(x) = \cos(jx) + \varepsilon a_{1j}\cos(2jx) + \varepsilon^2 a_{2j}\cos(3jx) + \dots, \quad \varepsilon = O(10^{-1}), \quad a_{ij} = O(1). \tag{3.4}$$

For $1 < \nu^2 < 3.25$, the attractor of Eq. (3.2) is the unimodal ($j = 1$) steady state (3.4). The limiting regimes for $3.25 < \nu^2 < 4.35$ are periodic orbits, and for $4.2 < \nu^2 < 5.63$ these are regular pulsation states. For $5.63 < \nu^2 < 10.75$, the attractor is the bimodal state (3.4) ($j = 2$). For $10.75 < \nu^2 < 13.5$, oscillatory and/or chaotic regimes occur, and for $13.5 < \nu^2 < 17$ the trimodal state (3.4) ($j = 3$) occurs [12, 14].

There is a qualitative correlation between the experimentally observable wave flows at small Reynolds numbers and representatives of calculated stationary periodic solutions that can be similar to nearly sinusoidal waves and wave formations with a higher content of Fourier components [10,11].

With increase in the value of ν, the number of unstable modes increases and regions of irregular behavior occur. For a rather high value of ν, the chaotic character of the dynamics increases sharply [13].

We turn to Eq. (3.1). For $k \neq 0$, the even function $J^*(k) \equiv kJ(k) > 0$ increases monotonically, and there are positive constants C_1 and C_2 such that $C_1k^2 < J^*(k) < C_2k^2$. By virtue of the properties of the function $J^*(k)$, the structures of the dispersion relations of Eqs. (3.1) and (3.2) are similar: only a finite number of longwave modes in the range $0 < n < k_0$ (a certain k_0 of the KS equation $k_0^2 = \nu^2$) can be linearly unstable; in the region of rather high wave numbers ($n > k_0$), the influence of dissipation turns out to be dominant, increasing rapidly by a power law. At the same time, for a given value of μ, with increase in the parameter A, the number of linearly unstable Fourier modes increases in comparison with the corresponding case $J \equiv 0$ ($k_0^2 > \nu^2$ for $\mu = \nu$). Figure 2 shows plots of the function $\gamma(k)$ for $\mu = 1$, $B = 1$, and $\sigma = 5$ for $A = 0$, 2, and 4 (curves 1–3).

By virtue of the specific properties of the dispersion relation of Eq. (3.1) one might expect that, for fixed $\mu = \mu_0$ ($B = B_0$ and $\sigma = \sigma_0$), an increase in the value of A (in a certain range) leads to approximately the same qualitative results as an increase in the parameter ν, beginning with the value equal to μ_0, for Eq. (3.2). The numerical calculations performed [the periodic solutions of Eq. (3.1) were investigated by the Galerkin method, and the initial data were specified in the form $\sum\limits_{j=1}^{m} \mu^2(\sin(jx) + \cos(jx))$] confirm this assumption.

Thus, for $\mu = 1$, $B = 1$, and $\sigma = 5$, the attractors of Eq. (3.1) are unimodal steady states for $0 < A < 3.5$ ($1 < k_0^2 < 3.26$) [as the value of A increases from 0 to 3.5, the content of Fourier-harmonics in the wave structure grows — the values of a_{i1} ($i > 1$) in (3.4) increase], regular traveling waves for $A = 4$ ($k_0^2 \approx 3.63$), ordered pulsation states for $A = 5$ ($k_0^2 \approx 4.39$), and bimodal pulsation states for $A = 8$ ($k_0^2 \approx 6.78$). For $\mu^2 = 7$, $B = 1$, and $\sigma = 5$, bimodal steady states form for $A = 0$ ($k_0^2 = 7$), and trimodal states form for $A = 12$ ($k_0^2 \approx 16.64$). It is useful to compare the values of k_0^2 in calculated examples with the above-stated boundaries of the range of the parameter ν^2 for attractors of various types of Eqs. (3.1) and (3.2).

The existence of limiting regimes of the type of j-modal states and regular traveling waves in the vicinity of $k_0^2 = 4$ can be substantiated analytically by the same method as for the KS equation [15].

Regimes that were ordered for $A = 0$ become chaotic for rather large values of A.

Thus, the analysis performed in the present paper shows that, within the framework of the adopted approximations, the presence of a rather strong electric field is a factor that increases the degree of irregularity of flow of a viscous dielectric liquid film.

At the same time, increasing the potential difference in the system considered for fixed parameters Re and We and spatial period of perturbations L appears to produce the same changes in the dynamics as an increase in the liquid flow rate in ordinary film flow provided that the KS model is adequate.

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